Project work

Solve the following problems and upload your solutions on the course site.

Problem 1 *Let $R = [0,4]\times[0,3]$ be the picture region with an $m\times n$ uniform partition, where $m = 3$ and $n = 4$. Consider the following line set:*

- *The lines l_1, l_2, l_3, l_4 are parallel to the vector $\underline{v}_1 = (0,1)$ and passing through the points $P_{1,1} = \left(\frac{1}{2}, 0\right)$, $P_{1,2} = \left(\frac{3}{2}, 0\right)$, $P_{1,3} = \left(\frac{5}{2}, 0\right)$, $P_{1,4} = \left(\frac{7}{2}, 0\right)$ respectively.*
- *The lines l_5, l_6, l_7 are parallel to the vector $\underline{v}_2 = (1,0)$ and passing through the points $P_{2,1} = \left(0, \frac{5}{2}\right)$, $P_{2,2} = \left(0, \frac{3}{2}\right)$, $P_{2,3} = \left(0, \frac{1}{2}\right)$ respectively.*
- *The lines $l_8, l_9, l_{10}, l_{11}, l_{12}$ are parallel to the vector $\underline{v}_3 = (1,2)$ and passing through the points $P_{3,1} = (-1,0)$, $P_{3,2} = (0,0)$, $P_{3,3} = (1,0)$, $P_{3,4} = (2,0)$, $P_{3,5} = (3,0)$ respectively.*

The line integrals of an unknown function f along lines l_k for $k = 1, 2, \ldots, 12$ are

$$\begin{array}{llll} m_1 = 1, & m_2 = 2, & m_3 = 2, & m_4 = 1, \\ m_5 = 2, & m_6 = 2, & m_7 = 2, & \\ m_8 = \frac{\sqrt{5}}{2}, & m_9 = 0, & m_{10} = \frac{3\sqrt{5}}{2}, & m_{11} = \frac{\sqrt{5}}{2}, \quad m_{12} = \frac{\sqrt{5}}{2}. \end{array}$$

(a) Find all the possible values $x_{i,j}$, such that the function g, which takes the constant value $x_{i,j}$ on the pixel $R_{i,j}$ for all $i = 1,2,3$ and $j = 1,2,3,4$, has the line integrals along the lines l_k equal to m_k for all $k = 1, 2, \ldots 12$.

(b) Which are the non-negative solutions?

(c) Find the non-negative solutions whose values are not larger than 1.

(d) Which are the binary solutions?

Problem 2 *Give a binary solution of the first problem if only the measurements along the lines l_1, l_2, l_3, l_4 and l_5, l_6, l_7 are known. This is equivalent to find binary matrix of size 3×4 with the row sum vector $R = (2,2,2)$ and column sum vector $S = (1,2,2,1)$. Is the solution unique?*

Problem 3 *Give a binary solution of the first problem if only the measurements along the lines l_1, l_2, l_3, l_4 and $l_8, l_9, l_{10}, l_{11}, l_{12}$ are known. Use the network flow algorithm.*